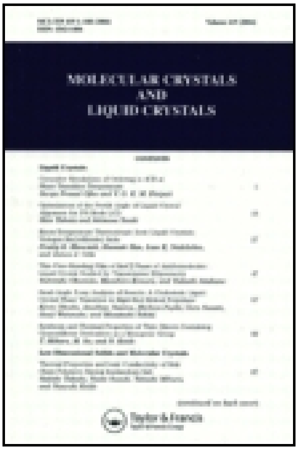

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General Equation of Director Alternating Azimuth Motion in a FLC Cell and Electrooptical Applications

Maxim Gorkunov [a], Sergei Pikin [b] & Elena Pikina [c]

[a] Moscow State Engineering Physical Institute, Kashirskoe shosse 31, 115409, Moscow, Russia

[b] Institute of Crystallography Russian Academy of Sciences, Leninskii prosp. 59, 117333, Moscow, Russia

[c] Oil and Gas Research Institute Russian Academy of Sciences, Leninskii prosp. 63/2, 117917, Moscow, Russia

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General equation of director alternating azimuth motion in a FLC cell and electrooptical applications

MAXIM GORKUNOV[a], SERGEI PIKIN[b] and ELENA PIKINA[c].
[a] Moscow State Engineering Physical Institute, Kashirskoe shosse 31, 115409 Moscow, Russia; [b] Institute of Crystallography Russian Academy of Sciences, Leninskii prosp.59, 117333 Moscow, Russia; [c] Oil and Gas Research Institute Russian Academy of Sciences, Leninskii prosp.63/2,117917Moscow, Russia

Repolarization kink motion in FLC cell is studied. The effects of the anisotropy energy time dependence and cell inhomogeneity are investigated.

Keywords: thin film ferroelectrics; kink switching; liquid crystals

INTRODUCTION

Director switching processes in FLC cell with high spontaneous polarization were consistently explained with the kink switching model[1-4]. This model assumes that, at the application of the electric field **E**, which is directed oppositely to the spontaneous polarization **P**, small areas of the cell are reoriented first, and then a sharply inhomogeneous perturbation with a small width runs through the initialy homogeneous film.

Since this repolarisation mechanism leads to the special form perturbation of the dielectric tensor, it affects the external field frequency dependence of the scattered light intensity in the electrooptical cell measurements, giving rise to the quasiresonant light scattering[3-4].

Here we present the generalization of previously developed theoretical model with the effects of harmonic time dependence of the heterogeneous energy of anisotropy taken into account. The kink motion in the inhomogeneous smectic layer is also investigated for some simpliest cases of local film defects.

EFFECTS OF THE HARMONIC TIME DEPENDENCE OF THE ANISOTROPY ENERGY

The equation of the director azimuth motion in the FLC cell under the action of the alternaiting external electric field $E = \widetilde{E}\cos(\omega t)$ can be written in the form:

$$\Xi \cos \nu\tau \sin\varphi - \frac{1}{2}\left[1 + V\cos^2 \nu\tau\right]\sin 2\varphi + \frac{\partial^2 \varphi}{\partial s^2} = \frac{\partial \varphi}{\partial \tau} , \quad (1)$$

where $\Xi = \frac{\mu \widetilde{E}}{U_0 \theta}$, $s = \frac{y}{\eta}$, $\tau = t\frac{U_0}{\gamma}$ and $\nu = \omega\frac{\gamma}{U_0}$ are the dimensionless field amplitude, coordinate, time and frequency correspondingly, μ is the piezoelectric modulus, θ is the tilt angle in the smectic C, γ is the viscosity coefficient, K is the elastic modulus and $\eta = \sqrt{\frac{K}{U_0}}$ is the characteristic spatial scale of the FLC. The energy of anisotropy is assumed to be of the form:

$$U = U_0 + \varepsilon_a E^2 \quad (2)$$

where ε_a is the dielectric permeability in the smectic plane, so that the parameter

$$V = \frac{\varepsilon_a \widetilde{E}^2}{U_0} \tag{3}$$

is rather small. Since it is impossible to write the exact solution of Eq.(1), this smallness can be used for the calculation of the approximate kink profile.

The total neglect of V containing term turns Eq(1) into the simplified equation of the kink motion, which was already explored [1–4]. Its solution takes the form

$$\varphi_0 = \arctan \frac{1}{\sinh\left[s - s_0 - (\Xi / \nu) \sin(\nu\tau)\right]} \tag{4}$$

and describes the oscillating kink motion with amplitude $\frac{\Xi\eta}{\nu}$ which is much greater than the kink width.

The contribution of kink perturbations to the dielectric tensor yields the quasiresonant light scattering effect [2–4], i.e., the intensity of scattered light has a maximum when this amplitude is equal to the cell thickness d:

$$\nu_{ex} \approx \frac{\Xi\eta}{d} \tag{5}$$

In the next approximation, the small value of V leads to the small correction to the kink profile function:

$$\varphi = \varphi_0 + \varphi_1 \ , \tag{6}$$

which can be found from the linear equation

$$\Xi \cos\nu\tau \tanh R \,\varphi_1 + \left(\cosh^{-2} R - \tanh^2 R\right)\phi_1 + \frac{\partial^2 \varphi_1}{\partial s^2} - \frac{\partial \varphi_1}{\partial \tau} = V \cos^2 \nu\tau \frac{\sinh R}{\cosh^2 R},$$

where (7)

$$R = s - s_0 - \frac{\Xi}{\nu} \sin(\nu\tau).$$

For the external field inducing the kink oscillations with a large amplitude, we assume that $\Xi >> 1$. Thus, we obtain:

$$\varphi_1 = \frac{V}{E} \cos \nu\tau \, \frac{\log \cosh R}{\cosh R} \; . \tag{7}$$

The corresponding scattered light intensity in the case of the same polarizations of scattered and incident light is propiotional to the integral

$$I_p = \int_0^{2\pi/\nu} dt \int_0^{d/\eta} ds \left[\cos^2 \varphi(s,\tau) - \cos^2 \varphi(s,0)\right]^2 \tag{8}$$

The contribution of small correction φ_1 can be written as

$$\Delta I_p = 2\frac{V}{\Xi} \int_0^{2\pi/\nu} dt \int_0^{d/\eta} ds \left[\frac{\varphi_1(t)\sinh R(t)}{\cosh^2 R(t)} - \frac{\varphi_1(0)\sinh R(0)}{\cosh^2 R(0)}\right] \left(\tanh^2 R(t) - \tanh^2 R(0)\right) \tag{9}$$

Finally, we obtain: (10)

$$\Delta I_p = 2\frac{V}{\Xi} \int_0^{2\pi/\nu} dt \int_0^{d/\eta} ds \frac{\sinh s \, \log\cosh s}{\cosh^3 s} \left(\tanh^2\left(s - \frac{\Xi}{\nu}\sin \nu\tau\right) - \tanh^2 s\right).$$

Since the $s \leq 1$ area is only sufficient in this integral, it does not depend on cell thickness d which is usually much larger than η. That is why the integral (10) frequency dependence is quiet similar to that of the quantity

$$\left\langle \xi^2 \right\rangle = \int_0^{2\pi/\nu} dt \int_0^1 ds \left(\tanh^2\left(s - \frac{\Xi}{\nu}\sin \nu\tau\right) - \tanh^2 s\right) \; , \tag{11}$$

which is in fact the change of the polarization component squared due to the kink motion in the extra thin cell with $d=\eta$. In particular it is known that $\left\langle \xi^2 \right\rangle$ has a weak maximum at the frequency value

$$\nu_{ex}^{(1)} \approx \Xi \quad , \tag{12}$$

so the intensity correction ΔI must also possess a weak maximum. In the case of other types of polarization, the scattered light intensity finds out to have the same main features, so its frequency dependence is of the same form.

KINK SWITCHING IN THE INHOMOGENEOUS FLC CELL

For the qualitative investigation of repolarization in the inhomogeneous FLC cell the time dependence of anysotropy energy can be neglected, thus a following class of nonlinear equations of motion, must be concidered:

$$\left(\Xi \cos \nu\tau + \sum_i \frac{\partial^2 u}{\partial s_i^2}\right) \sin\varphi - \frac{1}{2}\sum_i \left(\frac{\partial u}{\partial s_i}\right)^2 \sin 2\varphi + \sum_i \frac{\partial^2 \varphi}{\partial s_i^2} = \frac{\partial \varphi}{\partial \tau} \quad , \tag{13}$$

where $u = u(\mathbf{s})$ is an arbitrary function and $\varphi = \varphi(\mathbf{s}, t)$.

If it is assumed that at the time $\tau = 0$ (the time at which the field is switched on) such a region exists inside the FLC, bounded by the line $u(\mathbf{r}) = u_0 = \text{const}$,.and that inside it we have $\varphi(\mathbf{s}, \tau = 0) = 0$ and outside it $\varphi(\mathbf{s}, \tau = 0) = \pi$ or *vice versa,* and the width of the transition region is much smaller than the characteristic dimensionless film parameter s , then it can be easily seen that Eq.(13) has the exact solutions:

$$\varphi^{\pm} = \pm \arctan \frac{1}{\sinh\left[u(\mathbf{s}) - u_0 - (\Xi / \nu)\sin(\nu\tau)\right]} \tag{14}$$

The solutions (14) describe the motion of isolated orientational kinks of the azimuthal angle $\varphi(\mathbf{s}, \tau)$. For example, for $\nu = 0$ they describe the motion of a kink in a constant field along the s axis (subscripts 1 and 2 correspond to opposite dirrections motions):

$$\varphi_1^{\pm}(s,\tau) = \pm\arctan\frac{1}{\sinh\left[s - s_0 - \Xi\tau\right]}, \qquad (17)$$

$$\varphi_2^{\pm}(s,\tau) = \pm\arctan\frac{1}{\sinh\left[s - s_0 + \Xi\tau\right]},$$

with velocity $\mathrm{v} = \pm\Xi$, and the front plane perpendicular to the directon s.

The first term of the left -hand side of Eq.(13) describes the linear interaction between the spontaneous polarization and the effective field containing a homogeneous part Ξ (the external field) and some internal field contribution due to the electric properties of LC layer. The second term, quadratic in the polarization, describes the anisotropy energy influence on the motion of the spontaneous polarization vector; here the anisotropy energy density can be localy inhomogeneous, thancs to the film surface, domain walls, and structural deffects. The third term gives the usual contribution of the orientational elasticity energy. The right-hand side of Eq.(13) describes the contribution of the orientational viscosity and the corresponding moment of the friction forces.

Eq.(13) makes it possible to estimate the influence of inhomogeneity of the FLC on the repolarization processes. A few examples are given below.

1) Let the anisotropy energy density in the smectic layer plane (s_1, s_2) in the vicinity of some point defect vary quadratically , while the effective field is homogenious and constant, i.e.,

$$\left(\frac{\partial u}{\partial s_1}\right)^2 + \left(\frac{\partial u}{\partial s_2}\right)^2 = 4\left(s_1^2 + s_2^2\right),$$

$$\frac{\partial^2 u}{\partial s_1^2} + \frac{\partial^2 u}{\partial s_2^2} = \mathrm{const} \qquad (18)$$

In this case the functions $u(s_1, s_2)$ may take the following forms:

$$s_1^2 \pm s_2^2 \quad , \qquad\qquad 2s_1 s_2 . \tag{19}$$

Depending on the shape of the line $u(s_1,s_2)=u_0$ describing the surface of constant phase $\varphi=\pi/2$ at the initial time $t=0$, the motion of the phase front $\varphi(s_1,s_2,\tau)=\pi/2$, i.e. an orientational kink, can take various forms over the course of the time. According to the Eqs.(17) and (19), the corresponding fronts can have the forms of circles, parabolas, and hyperbolas, i.e., the following proposhional dependances are possible:

$$s_1^2 \pm s_2^2 \propto \tau \qquad\qquad 2s_1 s_2 \propto \tau . \tag{20}$$

2) Let the anisotropy energy density fall off exponencially according to the law $\exp(-2s_1)$ in some dirrection s_1 in the smectic layer, for example, with depth into the film away from a planar defect (the film surface), and let the effective field be homogenious and constant , i.e.,

$$\left(\frac{\partial u}{\partial s_1}\right)^2 + \left(\frac{\partial u}{\partial s_2}\right)^2 = e^{-2s_1}, \qquad \frac{\partial^2 u}{\partial s_1^2} + \frac{\partial^2 u}{\partial s_2^2} = 0 . \tag{21}$$

In this case the functions $u(s_1,s_2)$ may take the forms:

$$e^{-2s_1} \sin s_2 \ , \tag{22}$$

wich lead to wedge-shaped fronts, where the wedges are arrayed periodically along an isolated surface (along the s_2 axis) with period equal to the effective penetration depth of the front along the s_1 axis.

These simple examples, of course, do not exhaust the shapes of kink motion in a constant field. It is clear that the shapes and velocities of such fronts depend strongly on the local properties of FLC films. In the case of variable field Ξ action the solution (14) shows that the kinks execute oscillatory motion relative to some initial position with frecuency ν and amplitude propotional to $\tilde{\Xi} / \nu$.

CONCLUSIONS

Thus, the time dependence of the anisotropy energy leads to the existence of rather weak peak in the scattered light itensity frequency dependence. Its location is not influenced by the cell thickness and is situated in the relatively high frequency region. The cell inhomogeneity in the vicinity of local defects affects the form of the kink fronts, which strongly depends on local FLC properties.

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